COUNT Giordano Riccati of Treviso (1709–1790), though relatively little known to music historians, was a noteworthy theorist of harmony and the greatest Italian physicist in the field of acoustics. From his correspondence preserved at the Biblioteca Comunale in Udine we learn that already in 1740 he was working on his Delle corde ovvero fibre elastiche ('Of elastic strings or fibres'), which he eventually published in 1767. In the preface to this book we read:

The pleasure of music has drawn me to devote a fair amount of thought to those solid and fluid strings of which most musical instruments are made, and having at various times written eight chapters on this subject, I have recently gone through them page by page and revised them here and there so that with less embarrassment they can [now] be presented to the public.1

Riccati’s thought as a physicist extends well beyond the scope of a musician’s competence, but in chapters 5 and 6 the musician can find some very useful information on the ‘solid and fluid strings’ of harpsichords, violins and organs. In this article I shall analyze from an organological point of view the contents of chapter 6, which is entitled: ‘Of the dimensions which should be assigned to the strings of an instrument and to organ pipes, in order that they may render sounds of equal strength, and pleasing [to the ear]’2

According to Riccati, the area $S$ of the cross-section of a string of length $L$ will fall within the following limits (of which the second had already been posited by Leonhard Euler):

$$S = \text{constant} \quad S \propto L$$

Psycho-physical laws were as yet a thing of the future, so Riccati calculated the ‘best’ value of $S$ by assuming that the loudness of a vibrating string is proportional to its kinetic energy. (According to modern theory it is proportional to the logarithm of the kinetic energy, and also depends on the pitch frequency.) The results set out in chapter 6 are garnished with documentary evidence about musical instruments

*Much of this article is drawn from my forthcoming Acustica accordatura e temperamento nell’Illuminismo veneto.
in the Veneto which scholarship has overlooked but which is of unique historical value.

THE HARPSICHORD

Developing his assumptions mathematically, Riccati found that on a harpsichord, the cross-section \( S \) of the strings, ‘in order to render sounds of equal strength, and pleasing’, should be proportional to the geometric mean of the two limiting cases shown above. So \( S \propto \sqrt{L} \), and hence \( D \propto L^{1/4} \).

To verify this rule Riccati examined a ‘harpsichord built by Vito de’ Trasuntini in 1559’, measuring the strings for two Cs three octaves apart (p. 135). From the lengths given by Riccati one can deduce that they were for \( c \) and \( c'' \) of an 8’ instrument. (Evidently he did not measure the C string because in the lowest octave, as is well known, the curve of the bridge changes abruptly and quite abandons harmonic proportion among the string lengths.) For each of the two strings Riccati measured the vibrating length in ‘inches of the Venetian foot’ and, with a jeweller’s balance, the weight in Venetian grani of five feet of string. More than once in this work Riccati specified that the Venetian pound which he used as his unit of weight was equal to twelve ounces and amounted to \( 12 \times 576 \text{ grani} \): this is undoubtedly the 301.3-gram pound which was at that time the standard of weight among Venetian jewellers and pharmacists. Unfortunately, Riccati did not specify the material of which the strings were made; so in the following calculations I have used ‘Db’ and ‘Ds’ to refer to the diameters as they would be for brass \( (8700 \text{ kg/m}^3) \) and steel \( (7900 \text{ kg/m}^3) \) respectively. I use ‘\( W \)’ for the weight of the sounding length of string:

\[
\begin{align*}
\text{for } c & : L=32\frac{5}{6} \text{ (949.5mm)}; W=20 \text{ (=0.872g); } (Db=0.271, Ds=0.285\text{mm}) \\
\text{for } c'' & : L=4\frac{3}{4} \text{ (123mm)}; W=6 \text{ (=0.262g); } (Db=0.149, Ds=0.156\text{mm})
\end{align*}
\]

Since \( S:S'' \) (\( =W:W''=3.3 \)) was not very far from \( \sqrt{L:L''} \) (\( =2.8 \)), Riccati concluded that his rule was adequately confirmed by practice. If we assume that the string lengths would double in each octave, then by Riccati’s rule the diameters should have the ratio \( 1:2^{1/4} \) (\( \approx 1:1.19 \)). Bearing in mind also that from brass to steel the diameters would differ by only some 5%, we can conclude that on Trasuntino’s harpsichord the diameters were, approximately:

\[
\begin{align*}
c &= 0.28 \quad (c'=0.23) \quad (c''=0.19) \quad c'''=0.15 \text{ mm}
\end{align*}
\]

According to Riccati’s measurements \( L''':L \) amounted to 1:7.7, or slightly less than the ‘harmonic’ ratio 1:8. He explained this as follows:
In all the harpsichords and spinets that I have examined, when I have compared the lower strings with the higher ones I have found them to be somewhat shorter than the ratio of the vibration frequencies would require. From this it follows that the lower strings, on account of their thickness, are a little less tense than the higher strings. I believe that the practitioners do this because the hole through which the strings are passed [in the process of drawing them] compresses and strengthens thin strings more than [it does] thick ones, so the latter cannot bear the tension of a force quite proportional to their [greater] thickness. There is also the danger that the lower strings might be broken in the process of wrapping them around the pin, which flattens them. On the other hand, when they are protected from these risks and strung to the proper tension, they last a long time without breaking. The higher strings on the contrary do not last very well and break easily.  

(This last inconvenience has not been completely eliminated even today, partly because some harpsichord makers use diameters greater than 1/4 mm for the higher strings.) Riccati also showed (p. 138) that when the string is plucked, the shorter portion is disturbed more than the longer one: hence the string is more likely to break at the tuning pin than at the hitch pin.

Riccati concluded his discussion of harpsichord strings by saying that his rule should not be considered unalterable:

Experience together with theory has taught practitioners the suitable thicknesses for the lower and higher strings, and these may be, within certain discrete limits, done by approximation rather than exactly. When two strings of congruent thicknesses are put on an instrument, there remains another means — the force of the quills — of equalizing the strength of the sounds. When tuners play two strings at once, they understand exactly which quill should be increased or diminished in force in order that the two lower and higher sounds may make an equal impression on the ear.

The diameters which I have inferred from Riccati's measurements may seem rather small, but the relevant data published by Michael Thomas in 1971 tend to confirm them. Here are the string-gauge numbers shown on two late 16th-century Venetian instruments described by Thomas, and the equivalent values in millimetres as reckoned by Thomas himself and (in parentheses) by Kenneth Bakeman:

1. 'School of Trasuntino' (8' instrument, range C−f''')

<table>
<thead>
<tr>
<th>c</th>
<th>c'</th>
<th>c''</th>
<th>c'''</th>
<th>d'''</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>8</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>0.34</td>
<td>0.31</td>
<td>0.24</td>
<td>0.16</td>
<td>0.14 mm</td>
</tr>
<tr>
<td>(0.34)</td>
<td>(0.31)</td>
<td>(0.25)</td>
<td>(0.19)</td>
<td>(0.17) mm</td>
</tr>
</tbody>
</table>
2. 'Trasuntino' (4' instrument, range C–c'')

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>c</th>
<th>c'</th>
<th>c''</th>
<th>c'''</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>0.42</td>
<td>0.28</td>
<td>0.22</td>
<td>0.19</td>
<td>0.14 mm</td>
<td></td>
</tr>
<tr>
<td>(0.42)</td>
<td>(0.28)</td>
<td>(0.23)</td>
<td>(0.21)</td>
<td>(0.17) mm</td>
<td></td>
</tr>
</tbody>
</table>

Thomas also found on a harpsichord of a certain 'Franciscus, priest of Rimini' some old fragments of string with the following diameters:

c : 0.30

c' : 0.23

c'' : 0.14 mm

He found the same diameters in bits of iron or soft steel strings on a 'small Neapolitan instrument' (on top of which someone had later put brass strings of slightly greater diameter). According to Thomas, 'The conclusion from the data so far must be that strings at least as thin as .0055 in. (=0.14 mm) were used in Italy' in the 16th and 17th centuries. German makers used thicker strings, particularly in the treble.9

I might mention that in chapter 1 of Delle corde ovvero fibre elastiche Riccati described some experiments he had done with a brass string, from which it is possible to deduce10 that its elastic modulus was 8000 kg/mm²: thus the brass of that day was acoustically 'better' than now, given that today the modulus is some 9000 kg/mm². Indications of unit breaking stress (SB) for different metal wires can also be found in the same chapter (p. 18), where the results of measurements made by Joseph Sauveur some decades earlier11 are reported. From these may be deduced:

- steel: SB ≈ 10300 kg/cm²
- brass: SB ≈ 8500 kg/cm²
- copper: SB ≈ 4800 kg/cm²

Unfortunately Riccati did not say which metal the strings on his harpsichord were made of. (According to his theory this would not affect the choice of diameters.) I shall not try to settle this matter here, but I observe that:

i) In the experiments described in Riccati's book he used, exclusively, brass strings of small diameter.

ii) If the c''' string measured by Riccati was of brass, it bore some 5900 kg/cm² of tension (that is, some three semitones' worth less than the breaking stress, according to Sauveur's data shown above); if on the other hand it was of steel, the value would fall to 5350 kg/cm² (which is nearly six semitones' worth less than its breaking stress, a typical value for a 'slow' string). I have made these calculations knowing that Venetian pitch around the middle of the 18th century was 430–440 Hz for d'.12
iii) Nowadays it has been found that in order to work within reasonable limits of security, harpsichord strings should be stretched at most to two or three semitones' worth below their point of rupture.\textsuperscript{13}

iv) At the beginning of chapter 6, Riccati set out the premise that 'sonorous strings want to be stretched to such a degree that they would break if their tension were increased just a little more. . . . Stretching the strings a lot renders lively the palpitation of their smallest parts, [and] the sound consists mainly of this [palpitation]'. Also, as we saw above, he remarked that on his harpsichord the high strings 'do not last very well and break easily'.

So the balance of Riccati's evidence favours brass, at least for the instrument as it was in his day. (In which case the string used for the experiment described in chapter 1 would be for $c'$.)

In Appendix 1 I describe some remarks by Giambattista Doni (1695–1647) which suggest that Roman harpsichord makers (or at least some of them) at the time of Frescobaldi used steel strings for the higher notes, and copper for the very lowest.

THE ORGAN

Riccati adapted to 'fluid strings' the rule which he had devised for harpsichord strings. He began by showing that a harpsichord string of length $L$ 'pushes toward the ear' a mass of air proportional to $L^{3/12}$. But the mass of air that is set directly into vibration by a sonorous tube of length $L$ and diameter $D$ is proportional to the volume of the tube ($\propto D^2L$). Given, then, that the air waves produced by the pipe and by string are propagated at the same velocity, the condition which determines the ratio of their kinetic energies can be reduced to: $D^2L \propto L^{3/12}$; from which (and this conclusion is emphasized on p. xiv of the preface) Riccati obtained:

$$D \propto L^{19/24} \approx L^{3/4} \text{ (hence } S \propto L^{19/12} \approx L^{3/2})$$

(The same relation should hold for the mouth widths.) Riccati showed that since pipes an octave apart should have, theoretically, the ratio $L_1:L_2 = 2:1$, it follows that (pp. 144–45):

$$S_1:S_2 = 2^{3/2} = \sqrt{8},$$

(the geometrical mean between 2 and 4). Adopting a geometric division of the octave into twelve equal semitones, the diameters as one descends should thus increase as follows:

<table>
<thead>
<tr>
<th>C</th>
<th>B</th>
<th>Bb</th>
<th>A</th>
<th>Ab</th>
<th>G</th>
<th>Gb</th>
<th>F</th>
<th>E</th>
<th>Eb</th>
<th>D</th>
<th>Db</th>
<th>C</th>
<th>B</th>
<th>Bb</th>
<th>A</th>
<th>Ab</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L^{19/24}$</td>
<td>1.00</td>
<td>1.05</td>
<td>1.10</td>
<td>1.15</td>
<td>1.20</td>
<td>1.26</td>
<td>1.32</td>
<td>1.38</td>
<td>1.44</td>
<td>1.51</td>
<td>1.58</td>
<td>1.65</td>
<td>1.73</td>
<td>1.81</td>
<td>1.90</td>
<td>1.99</td>
</tr>
<tr>
<td>$L^{3/4}$</td>
<td>1.00</td>
<td>1.04</td>
<td>1.09</td>
<td>1.14</td>
<td>1.19</td>
<td>1.24</td>
<td>1.30</td>
<td>1.35</td>
<td>1.41</td>
<td>1.48</td>
<td>1.54</td>
<td>1.61</td>
<td>1.68</td>
<td>1.76</td>
<td>1.83</td>
<td>1.92</td>
</tr>
</tbody>
</table>
In the first case the doubling of the diameters takes place at about the minor 10th, in the second case at the major 10th.

Now $1: \sqrt{2}$ is the ‘diapason’ calculated by Töpfer in 1833 and used by most modern organ builders. Christhard Mahrenholz, evidently unaware of Riccati’s study, has attributed to G. A. Sorge (1773) the invention of the two relations under consideration. (Sorge actually proposed three kinds of diapason, in which the doubling of the diameters would take place at the major 9th, minor 10th and major 10th respectively.) Since theorists before that time had remained substantially tied down to the traditional ‘pythagorean’ scaling, Mahrenholz in his lucid analysis concludes that a ‘new era’ in the calculation of the diapason began with Sorge. This honour should be given rather to Riccati, who not only published his conclusions six years before Sorge, but also derived his calculations from objective principles of acoustics and not, like Sorge and Töpfer, from arbitrary arithmetical progressions.

Having found the rule, Riccati as usual turned to practice for its verification:

I measured very diligently with the aid of Signor Liberale Marcuzzi, the worthy organist of this Cathedral [of Treviso], the circumferences, proportional to the diameters, of two pipes which sounded [two] Fs three octaves apart. The organ of this church is a very fine instrument made by Urbano of Venice in 1420.

For an extra check Riccati gave also (p. 145) the circumference of the pipe for the octave above the lower F. From his measurements (in ‘lines of Venetian feet’) it is clear that the lower pipe corresponded to the first key (I indicate the diameter with ‘D’, the circumference with ‘C’):

$$
F : C_1 = 140 \text{ 'lines' } (= 337.5 \text{ mm}); \quad (D_1 = 107.5 \text{ mm})
$$
$$
f : C_2 = 84 \text{ 'lines' } (= 203 \text{ mm}); \quad (D_2 = 65 \text{ mm})
$$
$$
f'' : C_4 = 29 \text{ 'lines' } (= 70 \text{ mm}); \quad (D_4 = 22 \text{ mm})
$$

The corresponding diapason is very close to $2^{3/4}$. (For a perfect match one would have had to obtain $D_2 = 64$ and $D_4 = 22.5$.) Mahrenholz noticed that the ratio $1 : 2^{3/4} (\approx 1 : 1.68)$ approximates very closely to $3 : 5 (\approx 1 : 1.67)$, a ratio which he found in some 18th-century treatises. The maker of the organ described by Riccati probably followed such a procedure. In any case we can affirm that the so called ‘Töpfer ratio’ was already used by Italian organ builders in the 15th century.

**THE VIOLIN**

As each string on this instrument must serve for several notes, the first limit ($S = \text{constant}$) is imposed automatically in many instances. Yet if we intend that ‘the ear does not judge that [two equal notes] should
belong to one string rather than to another’, Riccati concluded that the cross sections of the three higher strings should correspond to their pitch frequencies, that is, to 3:2, thus yielding:

\[
\begin{align*}
\text{string:} & \quad D : A : E \\
\text{section:} & \quad 9 : 6 : 4 \\
\text{diameter:} & \quad 3 : 2.45 : 2
\end{align*}
\]

(We shall see presently why the G string was not included here.) He now proceeded to his usual practical verification:

With a jeweller’s scale I weighed three equally long sections, 1½ Venetian feet, of three violin strings, known as the tenor, the canto and the cantino. I did not weigh the lowest string because it is not of plain gut like the others, but is customarily overspun with a thin copper wire. If we recall that the cross-sections are proportional to the weights, we can see that the weights measured by Riccati confirm his theory almost perfectly. To calculate the diameters I have assumed that the specific gravity of the gut amounted to 1350 kg/m³ (tests which I have made on strings of different diameters and by different makers have yielded values ranging between 1300 and 1400):

\[
\begin{align*}
\text{D string} & = 15 \text{ grani} \quad (D = 1.09 \text{ mm}) \\
\text{A string} & = 10 \text{ grani} \quad (D = 0.89 \text{ mm}) \\
\text{E string} & = 6 \text{ grani} \quad (D = 0.69 \text{ mm})
\end{align*}
\]

For a perfect fit with Riccati’s rule the last weight should have been 6½ grani. Riccati, who was himself an amateur violinist, remarked that gut strings were quite imperfect and variable, and anyway ‘an expert player’ had told him that his (Riccati’s) E string was a little too thin. Riccati’s strings were somewhat thicker than those used by ‘baroque’ violinists today. Luigi Rovighi and Enrico Gatti have informed me that they use the following diameters:

\[
\begin{align*}
\text{D} & \quad \text{A} \quad \text{E} \\
\text{Rovighi:} & \quad 1.05 \quad 0.67 \quad 0.52 \text{ mm} \\
\text{Gatti:} & \quad 1.00 \quad 0.70 \quad 0.55 \text{ mm}
\end{align*}
\]

We should bear in mind, however, that already before 1743 in the Veneto, Tartini had introduced thicker violin strings. Count Gianrinaldo Carli wrote to Tartini in a letter of August 21, 1743 (from Venice to Padua): ‘You recognized that it was necessary to thicken the strings of the violin and lengthen the bow, as you have [indeed] done, so that the vibrations might be more regulated and the sound come out sweeter and more susceptible to vibrations. . .’. David Boyden has confirmed that ‘violin strings in Italy, where a full and strong tone was the ideal, were regular ropes compared to those used in France’. 

26
For harpsichord strings we sometimes have contemporary gauge numbers. Very little such direct evidence is available for violin strings, but a clue can be found in Jérôme de La Lande’s *Voyage en Italie* (1769), which describes the ‘method followed at Naples for the manufacture of violin strings’: ‘they put together only two strands of gut (from a 7- or 8-month-old lamb) for the little strings of the mandolin, three [strands] for the first string of the violin, seven for the last’.22 This would suggest a ratio between the diameters of the G and E strings of some $\sqrt{7:3} = 1.53$. Between the same two strings Riccati’s rule (which of course he did *not* consider applicable to the G string) would yield 1.84. Sébastien de Brossard in his early 18th-century manuscript treatise said of the G string: ‘if it is merely of gut it must be at least twice as thick as the D string, but if it is overspun with silver it is only a little thicker than the D string’.23 Riccati’s G string was overspun, so we might expect it to have been ‘only a little thicker’ than $1\frac{1}{2}$ times the thickness of the E string. Hence the value which we have deduced from La Lande’s description (1.53) is quite satisfactory.

In Appendix 2 I outline some additional documentary evidence for the diameters of 18th- and 19th-century violin strings.

**SUMMARY**

Riccati’s chapter yields the following organological information: i) the harpsichords of Vito Trasuntino had very thin strings. Their diameters, starting from a minimum of some 0.15 mm, doubled only after an interval of at least three octaves down. ii) One of the best Venetian organ makers of the 15th century (Antonio Dilmani)24 scaled the pipe diameters with a constant ratio equivalent to the modern ‘standard diapason’ of Töpfer. iii) Mid-18th-century Venetian violinists did indeed use rather ‘robust’ strings, for which Riccati furnishes us with precise diameters — data of unique value for the history of the violin.

**APPENDIX 1**

Giambattista Doni, Robert Smith and Carlo Pellegrini

on Harpsichord Strings

Two hitherto overlooked remarks by Giambattista Doni suggest that in Frescobaldi’s day Roman harpsichord makers used steel in the higher ranges, changing over to brass and then to pure copper for the lowest notes. In the first remark Doni, discussing around 1640 the difficulty of maintaining the pipes and strings of a claviorgan at the same pitch for the duration of an entire theatrical production, concluded: ‘. . . hence [claviorgani] could be quite useless, if it were not that this inconvenience can be remedied by tuning the
entire set of strings about a quarter-tone higher than the pipes in order that the whole instrument may come to its proper pitch when the air is warmed by the torches [which light the theatre]; and [indeed] if the higher strings did not also warm as much, either because they are made of steel or because they are so thin, it would be necessary to retune many of the notes. . . . '25 Doni indicated which metal the lowest strings were made of in a letter of August 7, 1638, from Rome to Father Mersenne at Paris, referring to the 'practice of our harpsichord makers, who ordinarily use, for the last notes in the bass, some strings of red copper, because those of brass do not have a long enough measure to render the required tone'.26

In 1759 Robert Smith referred indirectly to the practice of changing over to copper in the lowest range: for certain of his experiments he said that he used a 'copper wire, commonly used for some of the lowest notes on the harpsichord'. From his data one may deduce that the diameter of the string in question was 0.563 mm,27 that is, equal to gauge 11 according to Bakeman's table.28 The credibility of this conclusion is indicated by the fact that copper strings of the same gauge number were used, for example, in the lowest range (GG–BB) of a harpsichord described by James Talbot around 1698.29

On the other hand Carlo Pellegrini at Rome in 1665 said that the harpsichord normally had 45 brass strings (ex oricalcho confectas),30 and that: 'The [8'] spinetta is a very notable instrument, of the same type as the harpsichord, having the same number of strings as the latter; but its sound is not produced by brass as on the harpsichord, but by copper or steel. It is not sonorous like the harpsichord, being of smaller dimensions. Sometimes a red cloth is placed upon its strings in order that it may emit a more submerged sound'.31

APPENDIX 2

Plessiard's Rule for Violin Strings

A little more than a century after Riccati another amateur violinist, the French engineer Plessiard, worked out a different mathematical rule for the string dimensions.32 His theory was, like Riccati's, garnished with experimental data, which give us useful information about the diameters favoured by French violinists around the middle of the last century.

After remarking that the first three strings are of plain gut, Plessiard said 'that the rapidity of execution on the violin, the frequent passing from one string to another and the good effect of double stops [all] require that the bow not encounter a greater resistance on one string than on another, that is, that the work [required] to draw sound from each of the four strings be the same. To this end the ratio of tensions between two adjacent strings should be equal to the square root of the musical interval between them, i.e. \( \sqrt[3]{2} \), given that on the violin this interval is a fifth'.33 Unlike Riccati's rule this does not take into account the intensity of sound of the individual strings; according to Plessiard the acoustical equilibrium of the instrument should be regulated by adjusting the sound post and the bar. In passing from one string to the next larger, the two rules yield:
Thus Plessiari's rule yields a greater variation in the diameters for the sake of a more uniform distribution of tension among the four strings.

Plessiari then calculated the tension which one set of strings answering to his rule would impose upon violins of three different makers (p. 204). He gave the average specific weight of the gut which he used (1500 kg/m³), the frequency of his a' (435 Hz) and the sounding length of the strings, so I have been able to calculate the string diameters:

<table>
<thead>
<tr>
<th></th>
<th>Plessiari</th>
<th>Riccati</th>
</tr>
</thead>
<tbody>
<tr>
<td>ratio of tensions:</td>
<td>(2/3)¹/² = 0.82</td>
<td>2/3 = 0.67</td>
</tr>
<tr>
<td>ratio of diameters:</td>
<td>(3/2)³/⁴ = 1.355</td>
<td>(3/2)¹/² = 1.225</td>
</tr>
</tbody>
</table>

These diameters approximate to those in use at the time: Plessiari later specified (p. 209) that 'gut E strings varied from 0.40 to 0.55 grams' per metre, which corresponds to a diameter of 0.63–0.73 mm. For the G string he said that violin makers used an E string overspun with 16-gauge silvered copper wire, the weight of which varied, according to the brand, between 0.110 and 0.139 g/m. Since the average density of this wire was, according to Plessiari, 8880 kg/m³, its diameter must have been some 0.13–0.14 mm.

These figures confirm that 19th-century French violinists had increased the thicknesses of their strings so that they approximated more closely to those used in the Veneto in the preceding century. Indeed the French strings were probably even thicker by now, if we are to believe what Fétis wrote in 1856 on the basis of information which he had obtained from J.-B. Vuillaume, the greatest French violin maker of the day: 'Twenty years ago the E string required a tension of 22 pounds [i.e. nearly 11 kg] to get up to pitch, and the other strings a little less; thus the total weight was some 80 pounds [more than 39 kg]. The pitch standard had gone up a semitone since 1734; instruments were equipped with more robust strings; and the angle which they made on the bridge was sharper: hence the necessity of changing the violins' bar.'³⁴ The data furnished in 1806 by the Abbot Sibire also suggest considerable diameters.³⁵ In the following table I give the tensions in pounds as Sibire furnished them, their conversion into kilograms (as done by Plessiari) and the corresponding diameters (which I have calculated by assuming that L = 33 cm and that &d’ = 415–435 Hz):

<table>
<thead>
<tr>
<th></th>
<th>G</th>
<th>D</th>
<th>A</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>lbs:</td>
<td>13</td>
<td>15</td>
<td>17</td>
<td>19</td>
</tr>
<tr>
<td>kg:</td>
<td>6.36</td>
<td>7.34</td>
<td>8.32</td>
<td>9.30</td>
</tr>
<tr>
<td>(total = 64)</td>
<td>(total = 31.32)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(mm:</td>
<td>—</td>
<td>1.45–1.38</td>
<td>1.03–0.98</td>
<td>0.73–0.70</td>
</tr>
</tbody>
</table>

Fétis also reported, on the basis of information from Vuillaume, that 'Tartini
found as a result of experiments carried out in 1734 that the tension of the four strings on his instrument amounted to 63 pounds. But as this does not specify the kind of pound, the distribution of the tension among the four strings, and their sounding length, one cannot draw from it any secure conclusions as to their diameters.

One last observation: we have seen that at the time of Brossard the gut core of the G string had about the same diameter as the D string, whereas a century and a half later it had been reduced to the thickness of the E string. This must have favoured the adoption of the overspun D string; indeed Plessiard remarked that if the G string alone is overspun, then the D string ‘finds itself between two strings that are thinner than it and [hence] sound more brilliant. One could overspin it like the G string, and this would render the sound of the violin more homogenous. Various isolated attempts to carry out this change have already been made’. In any case this tendency was certainly witnessed in the preceding century by Brossard and Laborde. Some performers today as well, for example Sonya Monosoff, feel it necessary to use an overspun D string even when they are playing a baroque-style violin.

APPENDIX 3
Serafino Di Colco’s Rule for Violin Strings
Riccati was not the first to work out a rule for violin strings: in 1690 another Venetian, Serafino Di Colco, had tried to correct the prevailing custom whereby violinists chose the strings for their instruments by ear. He postulated a perfectly uniform distribution of tension. Thus, the diameters should vary still more than in Plessiard’s case:

<table>
<thead>
<tr>
<th>Di Colco</th>
<th>Plessiard</th>
<th>Riccati</th>
</tr>
</thead>
<tbody>
<tr>
<td>ratio of tensions</td>
<td>1</td>
<td>0.82</td>
</tr>
<tr>
<td>ratio of diameters</td>
<td>3/2</td>
<td>1.355</td>
</tr>
</tbody>
</table>

Di Colco’s proposal appears to be purely theoretical, as he does not provide any experimental data.

Translated by Mark Lindley.

NOTES

1 ‘Il piacere della musica mi ha invogliato di spendere non poche meditazioni intorno alle corde solide, e fluide, colle quali essa compone la maggior parte de’ suoi stromenti; ed avendo in varj tempi otto schediasmi distesi sopra questa materia, li ho nuovamente ripigliati per mano, e ritoccati qua e là, onde con meno di verecondia possano al pubblico presentarsi’ (p. iii).

2 ‘Delle misure, che debbono assegnarsi alle corde d’uno stromento, ed alle canne d’organo, acciòchè rendano suoni del pari forti, ed aggradevoli’.

3 Horace Doursther, Dictionnaire universel des poids et mesures . . . (Antwerp, 1840; facsimile ed. Amsterdam, 1965), p. 233. There was also a heavier Venetian pound of 12 × 768 grani (= 477.05 g). Evidence that Doursther’s 301.3-gram estimate applies here can be found in Riccati’s article, ‘Delle vibrazioni sonore dei cilindri’ (Memorie di matematica e fisica della Società italiana,
vol. 1, Verona, 1782, pp. 515 and 519), which gives the exact weights and dimensions of two cylinders, one of steel and the other of bronze.

4 These densities are given by Dijlda Abbot and Ephraim Segerman in their ‘Strings in the 16th and 17th centuries’ GSJ xxvii, (1974), p. 58.

5 ‘In tutti i gravicembali, e le spinette da me esaminati, poste al paragone le corde gravi colle acute, le ho trovate alquanto più corte di quello che richiede la proporzione dei tempi delle loro vibrazioni. Da ciò deriva la conseguenza, che le corde gravi rispetto alle loro grossezze sono un po meno tese delle corde acute. Io credo, che i pratici così si adoperino, perché la trasfìla, per cui si fanno passare le corde, costipì e renda tenaci più le sottili delle grosse, dimostrandolì queste non possano tollerare la tensione di una forza alle loro grossezze precisamente proporzionale. Corrono inoltre pericolo le corde gravi di scavezzarsi, mentre si attorcigliano per attaccarle allo stromento, e si rivolgono intorno al perno, col mezzo del quale si stirano. Per altro sfuggiti i descritti rischi, e ridotte alla dovuta tensione, si conservano molto tempo senza spezzarsi. Le corde acute all’opposto, durano poco, e si rompono facilmente.’ (pp. 137–38).

6 ‘Non è arbitrario l’armare uno strumento con corde di qualunque grossezza, ed armato che sia l’usar forze a capriccio per far suonar esse corde. L’esperienza uniforme alla teorica ha insegnato ai pratici le convenienti grossezze delle corde gravi ed acute, le quali surrogando il prossimo all’esatto, ponno stare entro certi discreti limiti. Applicate allo strumento due corde di congrua grossezza, resta l’altro mezzo della forza delle penne, per interamente pareggiare il vigore dei suoni. Toccano gli accordatori nello stesso tempo due corde, comprendono esattamente quale penna debba accrescersi, o scemarsi di forza, acciocchè i due suoni grave ed acuto facciano nell’orecchio eguale impressione.’ (p. 139).


9 According to Michael Thomas (note 7, p. 71), German clavichord strings in the treble became no smaller than gauge 7. Friedemann Hellwig has, in effect, confirmed this (‘Strings and stringing: contemporary documents’, GSJ xxix, 1976, pp. 91–104). There were exceptions, however. An 18th-century manuscript annotation which I have come across on the next-to-last page of a volume containing two works of Andrea Werckmeister (Paris, Bibliothèque Nationale, Mus Rés 1747) prescribes the following gauges for an unfretted clavichord of range C–c’’’ (without C#):

<table>
<thead>
<tr>
<th>C</th>
<th>D#</th>
<th>F</th>
<th>G#</th>
<th>c</th>
<th>f</th>
<th>d’</th>
<th>g’</th>
<th>d”</th>
<th>a”–c’’’</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9–9</td>
</tr>
</tbody>
</table>

10 Pp. 20–22. Here are the data (in Parisian, the weights in Venetian units): 390 ‘lines’ of brass string weighed 7.5 grani (1 ounce = 576 grani); such a string (the diameter was 0.233 mm) was stretched first by a weight of two pounds; when the weight was increased by four pounds (to six), the string became 1⅛ ‘lines’ longer, and as much again by adding another four pounds.
11 According to Sauveur, strings of steel, brass and copper broke when stretched by weights amounting to 12000, 9000 and 5000 times, respectively, the weight of 40 inches (of the Parisian foot) of the string.

12 See my 'Il corista bolognese, secondo il rilevamento di V. F. Stancari' L'Organò xviii, 1980, p. 25.


15 A letter of July 2, 1753, from Francescanonì Vallotti to Riccati (Udine, Biblioteca Comunale, MS. 1027) implies that Marcuzzi was by then too old to play the organ — which in turn suggests that Riccati’s account was written a good many years before its publication.

16 'Misurai diligentissimamente coll’ajuto del Signor Liberale Marcuzzi valoroso organista di questa Cattedrale le circonferenze proporzionali ai diametri di due canne, che suonavano F fa ut, e si corrispondevano in triplo ottava. L’organo della nominata Chiesa è un’opera assai perfetta lavorata da Urbano da Venezia l’anno 1420.' (p. 143). I believe Riccati’s historical information was inexact. Renato Lunelli, Studi e documenti di storia organaria veneta (Florence, 1973), gives the following account of the cathedral organ at Treviso: in 1436 Niccolò d’Allemagna gave the church a new organ (p. 212); Antonio Dilmani, the son of the organ-maker Bernardo d’Allemagna, rebuilt it ex novo in 1481–83, and carried out further work between 1493 and 1500 (p. 180); in 1773 the instrument, after numerous restorations, was rebuilt by Gaetano Callido (p. 135). Lunelli also says (pp. 227–28) that Urbano of Venice died before 1518. It appears that Riccati measured pipes for which Dilmani was most likely responsible.

17 Marenholz, op. cit., pp. 41, 66 and 69. The same scaling appears to have been used by the Töpfer’s Roman contemporary, Francesco Pasquetti, who probably would not have known Töpfer’s treatise; see my ‘Rilievi tecnici sull’organo ’Franciscus Pasquetti Romae fecit A.D. 1836’ ’, Quaderni umbri, xvi, 2 (Terni, March 1983), p. 12.

18 ‘Colle bilancette dell’oro pesai tre porzioni equalmente lunghe piedi 1½ Veneziani delle tre corde del Violino, che si chiamano il tenore, il canto e il cantino. Tralasciando d’indagare il peso della corda più grave, perché questa non è comò l’altra di sola minugia, ma suole circondarsi con un sottile filo di rame.’ (p. 130).


20 The New Grove, s.v. ‘Violin’, p. 828. I imagine Boyden drew this conclusion from François Ragueneau’s Comparison between French and Italian music (1702); see apropos Boyden’s The history of violin playing from its origins to 1761 (London, 1965), p. 203.

21 For some exceptional violin gauge numbers see note 35.

22 ‘Méthode suivie à Naples pour la fabrication des cordes de violon... on ne met que deux boyaux ensemble pour les petites cordes des mandolines, trois pour la premiere corde de violon, sept pour la derniere.’ Journal de musique

23 ‘Si elle est simplement de boyau, elle doit estre du moins le double plus grosse que la 3e, mais si elle est toute filée d’argent elle n’est que très peu plus grosse que la 3e.’ Paris, Bibliothèque Nationale, Mus Res Vm8 c 1, fol. 12r.

24 See note 16.

25 ‘... dal che si potrebbero rendere del tutto inutili, se non fosse che ancora a questo pare che si possa rimediare, con accordare tutto il sistema delle corde un quarto di tuono in circa più acuto del sistema delle canne, che così poi riscaldata, che fosse l’aria dalle torce, col calore verrebbe alla sua giusta voce; e se le corde acute, o per essere d’acciaio, o per avere manco materia non calassero tanto, allora sarebbe necessario ritoccare molti tasti, per ridurlì interamente al suo sesto.’ Giovanni Battista Doni, ‘Trattato della musica scenica’, Opere, ii (Florence, 1763), p. 112. Doni died in 1647 and the ‘Trattato’ was written before 1642.

26 ‘... pratique de noz faiseurs de clavecins; lesquels mettent d’ordinaire aux dernières voix en bas quelque chordes de cuivre rouge, pource que celles de leton n’ont pas la mesure assez longue pour rendre le ton requis.’ Cornelis de Waard, ed., Correspondance du P. Marin Mersenne, vol. viii (Paris, 1963), p. 20. The use of brass in the bass of Roman harpsichords at that time is also confirmed by Pier Francesco Valentini: in one of his manuscripts (c. 1642–45) he gave instructions for making a monochord with a brass string six Roman palmi long, and remarked, ‘qual corda sarà della grossezza dei bassi di un cembalo ordinario’ (Monocordo et nova costituzione di musica . . . , p. 23 — Biblioteca Apostolica Vaticana, MS. Barberini lat. 4430).


28 Bakeman, note 8, p. 99.


30 Carlo Pellegrini, Museum historico-legale bipartitum, i (Rome, 1665), p. 15.


32 Plessiard, ‘Des cordes du violon’, Association française pour l’avancement des sciences. Congrès de Lille, 1874, 2e Groupe (Sciences physiques et chimiques), ‘Séance du 21 août 1874, pp. 192–220. I have not been able to find Plessiard’s first name; he is referred to as ‘Ingenieur en chef des Ponts et Chaussées en retraite’.

33 ‘... que la rapidité d’exécution sur le violon, que le fréquent passage d’une corde à une autre et le jeu sur double corde exigent que l’archet ne rencontre pas plus de résistance sur une corde que sur une autre, c’est-à-dire que le travail soit le même pour obtenir le son des quatre cordes. Pour cela le rapport des tensions de deux cordes consécutives doit être égal à la racine carrée de leur intervalle musical ou à $\sqrt{3}/2$, puisque pour le violon cet intervalle est
une quinte.' Op. cit., p. 196. Plessiard seems not to have known of Riccati's work. He found his rule by means of a formula for the work Q of the bow: $Q = kmtf$, where $k$ is a constant and $m$, $t$ and $f$ represent the string's mass, tension and vibration frequency.

34 'Il y a vingt ans, la chanterelle n'arrivait à son intonation qu'avec un poids de 22 livres, et les autres cordes un peu moins; la charge était donc d'environ 80 livres. Le diapason s'était élevé d'un demi-ton depuis 1734; on montait les instruments en cordes plus fortes, et l'angle qu'elles fasaient sur le chevalet était plus aigu: de là la nécessité de rebarrer les violons.' François-Joseph Féris, Antoine Stradivari luthier célèbre . . . (Paris, 1856), p. 92. The preface explains that the work is based on investigations which Vuillaume had carried out (in Italy as well as France): Vuillaume also edited the book. Féris employed the French pound (= 489.5 grams: H. Doursther, note 2, p. 228).

35 Sébastien-André Sibire, La chélonomie ou le parfait luthier (Paris, 1806), pp. 112–13. (The violin was tuned 'au ton de la flûte'.) According to Plessiard (op. cit., p. 213), Sibire's work was based upon 'notes of the luthier Lupot'. Albert Cohen, 'A cache of 18th-century strings' (GSJ xxxvi, pp. 37–48), describes a number 3 gauge 'Chanterelle de Violon p. le C. en Gosseque' (i.e. François-Joseph Gossec) of which the diameter, according to Cohen, is 0.70 mm. This matches the evidence from Sibire and from Riccati.


37 ' . . . se trouve alors entre deux cordes plus minces qu'elle et qui sonnent avec plus d'éclat. On pourrait la filer comme la quatrième, qui rendrait les sons du violon plus homogènes. Divers essais isolés de cette substitution ont déjà été faits.' Op. cit., p. 197.

38 Brossard, loc. cit.: 'if [the D string] is merely of gut it must be at least twice as thick as the A string, but if it is half-overspun with silver, as it almost always the case nowadays, it is no thicker than the A string'. Regarding Laborde (Essai sur la musique, 1780), see Eduard Melkus, Il violino (Florence, 1975), p. 27.

39 'Siano da proporzonarsi ad un Violino le corde, come si vede nella figura. Iconismo 2. figura 2. A.b.c.d. distese, e distirate da pesi vguali E.f.g.h. Se toccandole, ò suonandole con l'arco formeranno vn Violino benissimo accordato, saranno bene proporzionate, altrimenti conuerrà mutarle tante volte, sin tanto che l'accordatura riesca di quinta due, per due, che appunto tale è l'accordatura del Violino.' Serafino Di Colco, 'Lettera prima (Venezia, 7 Gennaro 1690)', Le veggie di Minerva nella Accademia de Filareti: Per il mese di Gennaio 1690 (Venezia, 1690), pp. 32–33. In the same letter he describes the 'Moderna accordatura di Clauicimbalo' ($\frac{3}{4}$-comma temperament, from Eb up to G#).

40 Plessiard's ratio is the geometric mean between those of Di Colco and Riccati.